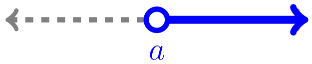
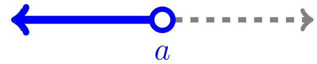
1.2: Domain and Range- Piecewise Defined

Different Types of Intervals

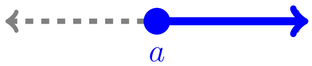
$x > a$ or (a, ∞)



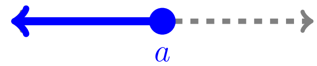
$x < a$ or $(-\infty, a)$



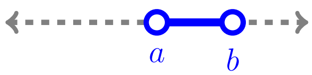
$x \geq a$ or $[a, \infty)$



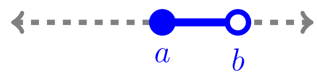
$x \leq a$ or $(-\infty, a]$



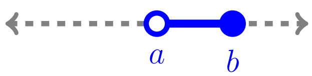
$a < x < b$ or (a, b)



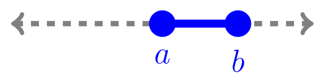
$a \leq x < b$ or $[a, b)$



$a < x \leq b$ or $(a, b]$



$a \leq x \leq b$ or $[a, b]$



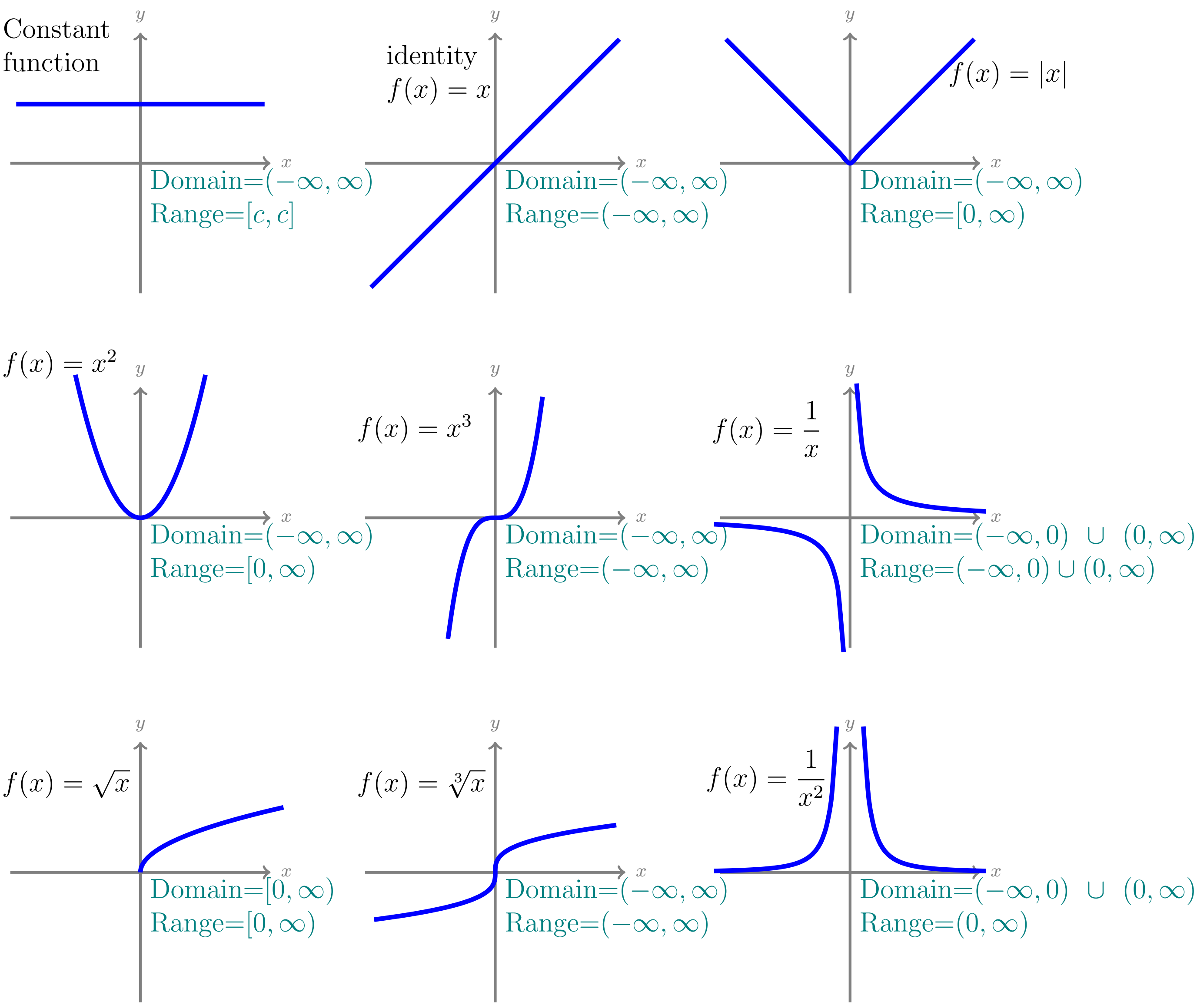
Solving Polynomial Inequalities

- Add or subtract to have all terms on one side with 0 on the other side of inequality. (It is preferred to have the polynomial on the greater side. That is, the inequality is being solved for > 0 or ≥ 0.)
- Find the roots of the polynomial.
- Make a number line with roots placed in the correct order. The roots partition the number line into open intervals and endpoints.
- Use test points to find the sign of each interval.
- All intervals which match the inequality sign are included in the solution.
- Check the equal signs to see if an end point is included.
- One of the applications of solving inequalities is to find the domain of radical and logarithmic functions.

Finding Domain of a Function Using Algebraic Rules

- **Exclude** the zeros of the denominator. (Chapter 1)
- **Exclude** all intervals that make under the square root negative. (This requires solving for an inequality.)
- **Exclude** what makes inside a logarithmic function negative or zero. Logarithmic functions will appear in Chapter 4.
- What remains of the number line is the domain.

Domain and Range of Well-known Functions



Graphing and Finding Values for Piecewise-defined Functions

- Find the cut-off input values.
- Using the cut-off values, divide the xy-plane into as many pieces as in the rule of the function.
- Graph each piece of the rule in its section.
- Additionally, if you need to find function value at some input value, find the interval that corresponds to the value. Then plug in the input value in the corresponding rule.

Absolute Value Function: Piecewise-defined

- $f(x) = |x| \implies f(x) = \begin{cases} x & \text{when } x \geq 0 \\ -x & \text{when } x < 0 \end{cases}$ (Check this fact by taking a sample value in each rule.)
- $|x-a| = \begin{cases} x-a & \text{when } x-a \geq 0 \\ -(x-a) & \text{when } x-a < 0 \end{cases} \implies |x-a| = \begin{cases} x-a & x \geq a \\ -x+a & x < a \end{cases}$

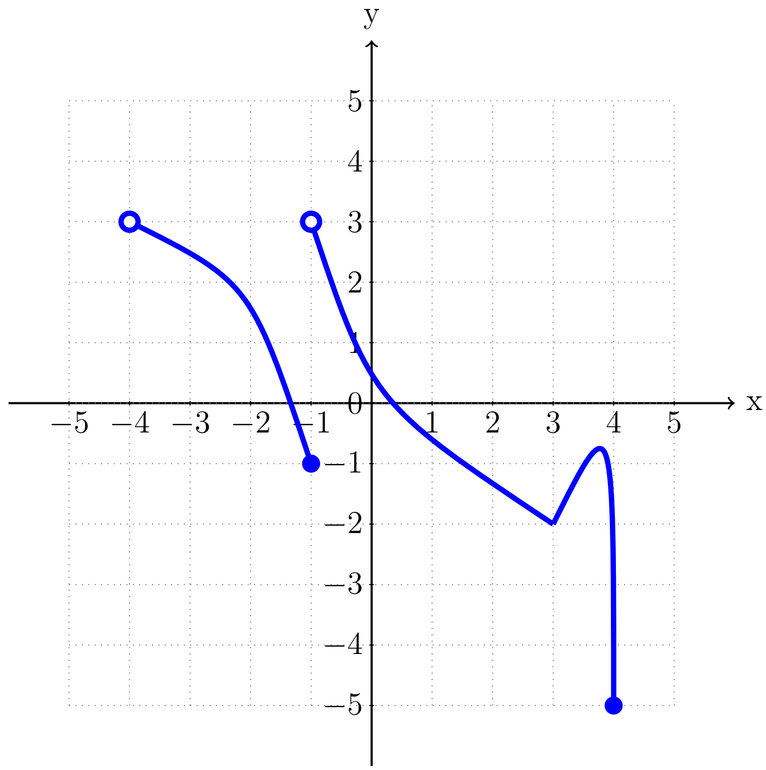
Review: A few Properties of Inequalities

$$a > b \implies a \pm c > b \pm c$$

$$a > b \underset{\text{If } c>0}{\implies} ac > bc.$$

$$a > b \underset{\text{If } c<0}{\implies} ac < bc.$$

1. The graph of function g is given below. Find the domain and the range of the function.



2. Find the domain of $f(x) = \sqrt{7-x}$.

3. What is the domain of $f(x) = x^2 - 4$.

4. Solve each of the following inequalities for x.

   (a) $-(x-11)(x+2) \geq 0$

   (b) $x^2+5x+6>0$

   (c) $3x^2-3x<2x^2-4$

   (d) $(x-2)^3(x-1)(x+11) \leq 0$

5. Solve $2<5x-1 \leq 11$ for x.

6. Find the domain of these functions.

   (a) $f(x)=\sqrt{-(x-5)(x+3)}$

   (b) $g(x)=\sqrt{(x-3)^3(x-1)(x+5)}$

   (c) $h(x)=\dfrac{1}{(x-5)(x+3)}$

7. (a) Solve the equation $x^2 - 6x - 7 = 0$

(b) Solve $x^2 - 6x - 7 < 0$ for x.

(c) What is the domain of $f(x) = \sqrt{x^2 - 6x - 7}$? Express your answer using interval notation.

8. Let

$$f(x) = \begin{cases} -x - 7 & \text{when} \quad x < -3 \\ x & \text{when} \quad -3 \leq x < 2 \\ 2x - 2 & \text{when} \quad x \geq 2 \end{cases}$$

(a) Find the value of function at following input values: $f(-7)$, $f(-4)$, $f(-3)$, $f(1)$, $f(2)$ and $f(3)$.

(b) Sketch a graph of $f(x)$, labeling at least 4 points.

(c) What is the domain of the function?

(d) Use the graph to determine the range of the function.

9. According to the nerd wallet blog, the 2022 Federal Tax Income Tax Brackets for Single Filler is

Tax rate	Taxable income bracket	Tax owed
10%	\$0 to \$10, 275	10% of taxable income
12%	over \$10, 275 to \$41, 775	\$1, 027.50 plus 12% of the amount over \$10, 275
22%	over \$41, 775 to \$89, 075	\$4, 807.50 plus 22% of the amount over \$41, 775
24%	over \$89, 075 to \$170, 050	\$15, 213.50 plus 24% of the amount over \$89, 075
32%	over \$170, 050 to \$215, 951	\$34, 647.50 plus 32% of the amount over \$170, 050
35%	over \$215, 951 to \$539, 900	\$49, 335.50 plus 35% of the amount over \$215, 951
37%	over \$539, 900	\$162, 718 plus 37% of the amount over \$539, 900

(a) Explain how \$1, 027.50 was calculated in tax bracket over \$10, 275 to \$41, 775.

(b) Explain how \$15, 213.50 was calculated in tax bracket over \$89, 075 to \$170, 050.

(c) Express the tax owned as a function of income in dollars. [1]

$$I(x) = \begin{cases} & \text{when } 0 \le x \le 10,275 \\ & \text{when } 10,275 < x \le 41,775 \\ & \text{when } 41,775 < x \le 89,075 \\ & \text{when } 89,075 < x \le 170,050 \\ & \text{when } 170,050 < x \le 215,951 \\ & \text{when } 215,951 < x \le 539,900 \\ & \text{when } x > 539,900 \end{cases}$$

[1] Do at least the first four brackets in class. This gives you an understanding of the need for piecewise defined functions

(d) Graph of the rate versus taxable income bracket is sketched below. We shaded the area under the graph, above the x-axis and between $x = 0$ and $x = 350,000$. Explain how the shaded area is related to the function in Part (c).

Tax rate: the blue step function